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UNIFIED COUNCIL OLYMPIADS

Unified International Mathematics Olympiad

UNIFIED INTERNATIONAL MATHEMATICS OLYMPIAD (UPDATED)

CLASS - 9
Question Paper Code : 4P104

KEY

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A	B	D	C	A	D	A	A	C	A
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C	B	C	D	B	B	B	B	A	A
21	**22**	**23**	**24**	**25**	**26**	**27**	**28**	**29**	**30**
D	C	A	C	C	D	B	B	D	C
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A,C,D	A,B,C	A,B	A,B,C	A,C,D	C	D	C	D	A
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C	C	C	B	D	B	C	D	B	Del

SOLUTIONS

MATHEMATICS - 1 (MCQ)

01. (A) $\sqrt{4a^2+ab^2+16c^2+12a-24bc-16ca}$ =

$\sqrt{(2a)^2+(3b)^2+(-4c)^2+2(2a)(3b)+2(3b)(-4c)+2(-4c)(2a)}$

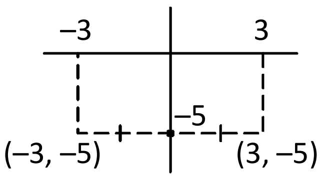
$=\sqrt{(2a+3b-4c)^2}$

02. (B) $x^2 + 2x + 1 - x^2 + 1 = 2x^2 + x - 2(x^2 + 3x + 2) + 20$

$2x + 2 = 2x^2 + x - 2x^2 - 6x - 4 + 20$

$7x = 16 - 2 \Rightarrow x = 2$

03. (D)



04. (C) $\Delta ADH \cong \Delta JIH$ [$\because$ ASA congrueny]

$\therefore$ Area of ΔADH = area of ΔJIH

$\therefore$ Shaded area : Total area = 1 : 3 = $\frac{1}{3}$

05. (A) Given $\frac{4}{3}\pi r^3 = \frac{1}{3}\pi \times 6 \times 6 \times 24$ cm3

r = 6 cm

06. (D) $a^2 + 6ab + 9b^2 + b^2 + 2bc + c^2 + 4c^2 - 16c + 42 = 0$

$(a + 3b)^2 + (b + c)^2 + (2c - 4)^2 = 0$

Sum of three perfect squares is zero then each term must be zero

$\therefore$ a = – 3b, b = – c, 2c = 4

a = 6, b = – 2, c = 2

$\therefore$ a – b + c = 6 + 2 + 2 = 10

07. (A) $9^{\frac{1}{3}}, 11^{\frac{1}{4}}, 17^{\frac{1}{6}}$

$9^{\frac{4}{12}}, 11^{\frac{3}{12}}, 17^{\frac{2}{12}}$

$\sqrt[12]{9^4}, \sqrt[12]{11^3}, \sqrt[12]{17^2}$

$x = \sqrt[12]{6561}$ $y = \sqrt[12]{1331}$ $z = \sqrt[12]{289}$

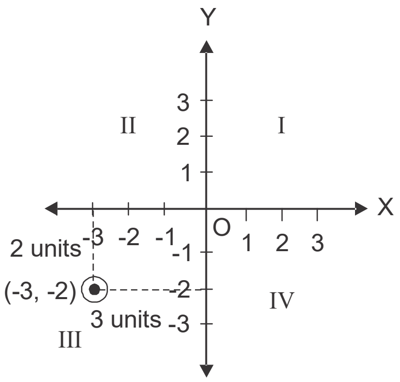
$\therefore x > y > z.$

08. (A) The perpendicular distance of a point from x-axis = 2 units.

The perpendicular distance of a point from y-axis = 3 units

Given, that the point lies in the III Quadrant

$\Rightarrow$ Both the coordinates of the point are negative.



$\therefore$ The required coordinates of the point are (–3, –2).

09. (C)

$$S = \frac{a+b+c}{2} = \frac{21m + 20m + 13m}{2} = \frac{54m}{2} = 27m$$

$$\Delta = \sqrt{S(s-a)(s-b)(s-c)}$$

$$= \sqrt{27 \times 6 \times 7 \times 14}\ m^2$$

$$= \sqrt{3 \times 9 \times 2 \times 3 \times 7 \times 2 \times 7}\ m^2$$

$= 3 \times 3 \times 2 \times 7\ m^2$

$= 126\ m^2$

10. (A) Given $2\not{\pi}r = 14\not{\pi}$ cm

$$r = \frac{14cm}{2} = 7cm$$

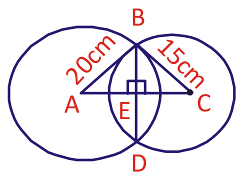
$\therefore$ Height = 2r = 14 cm

$$\text{Volume} = \pi r^2 h = \frac{22}{\not{7}} \times \not{7} \times 7 \times 14 cm^3$$

= 2156 cm3

$= \frac{2156}{1000}$ Litres = 2.156 Litres

11. (C) Given AB = 20 cm & BC = 15 cm



AC = 25cm and BD $\perp$ AC

In ΔABE, Let $AE = x cm \Rightarrow EC = (25 - x)$ cm

$BE^2 = AB^2 - AE^2 = (20)^2 - x^2 = 400 - x^2 \rightarrow$ (1)

In ΔBCE, $BE^2 = BC^2 - EC^2 = (15)^2 - (25 - x)^2$

$= 225 - (625 - 50x + x^2)$

$= 225 - 625 + 50x - x^2$

$= 50x - x^2 - 400 \rightarrow$ (2)

But eq (1) = eq (2)

$$400 - \not{x^2} = 50x - \not{x^2} - 400$$

$400 + 400 = 50x$

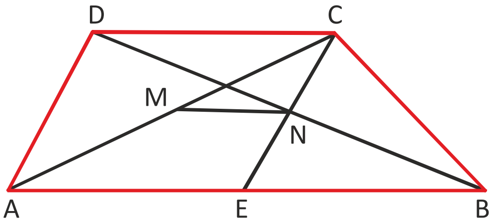
$$x = \frac{80\not{0}}{5\not{0}} = 16$$

$\therefore$ $BE2 = 400 - x^2 = 400 - 162 = 400 - 256 = 144$

$\therefore BE = \sqrt{144cm} = 12cm$

$\therefore$ BD = 2BE = 2 × 12cm = 24 cm

12. (B)



Const: Join CN and extend upto E

Proof: $\Delta CND \cong \Delta ENB$

[$\because$ ASA Congruency]

$\therefore$ CN = NE $\Rightarrow$ 'N' is the mid point of CE & CD = BE

In ΔACE, MN = $\frac{1}{2}$ AE = $\frac{1}{2}$(AB – BE)

= $\frac{1}{2}$(AB – CD) = 3 cm

13. (C) $\frac{1}{x} = \frac{1}{9+4\sqrt{5}} \times \frac{9-4\sqrt{5}}{9-4\sqrt{5}} = \frac{9-4\sqrt{5}}{9^2-(4\sqrt{5})^2}$

= $9 - 4\sqrt{5}$

$x + \frac{1}{x} = 9 + 4\sqrt{5} + 9 - 4\sqrt{5} = 18$

Squaring on both sides

$\left(x + \frac{1}{x}\right)^2 = 18^2$

$x^2 + 2 \times \cancel{x} \times \frac{1}{\cancel{x}} + \frac{1}{x^2} = 324$

$x^2 + \frac{1}{x^2} = 324 - 2 = 322$

14. (D) Given $(x – 1)^8 = a^8x^8 + a^7 x^7 + a^6 x^6 + ... + a^1x + a^0$

Given $(x – 1)$ is a factor of $f(x) = (x – 1)^8$

$\therefore$ f(1) = 0

$\therefore$ $f(1) = a^8(1)^8 + a^7(1)^7 + a^6(1)^6 + ... + a^1(1)^1 + a^0 = 0$

$\therefore$ $a^8 + a^7 + a^6 + a^5 + a^4 + a^3 + a^2 + a^1 + a^0 = 0$

15. (B) From options If x = 3 then

$\sqrt{3+1} + \sqrt{6+3} = 2 + 3 = 5$ (or)

Given $\sqrt{x+1} + \sqrt{2x+3} = 5$

Squaring on both sides

$\left(\sqrt{x+1} + \sqrt{2x+3}\right)^2 = 5^2$

$x + 1 + 2\sqrt{x+1}\sqrt{2x+3} + 2x + 3 = 25$

$2\left(\sqrt{x+1}\right)\left(\sqrt{2x+3}\right) = 25 - 4 - 3x$

$2\left(\sqrt{x+1}\sqrt{2x+3}\right) = 21 - 3x$

Squaring on both sides

$4(x + 1)(2x + 3) = (21 – 3x)^2 = 441 – 126x + 9x^2$

$4(2x^2 + 5x + 3) = 441 – 126x + 9x^2$

$8x^2 + 20x + 12 – 9x^2 + 126x – 441 = 0$

$–x^2 + 146x – 429 = 0$

$x^2 – 146x + 429 = 0$

$x^2 – 143x – 3x + 429 = 0$

$x(x – 143) – 3(x – 143) = 0$

$\therefore$ $(x – 3)(x – 143) = 0$

x = 3 (or) x = 143 but x = 143 does n't satisfy the given question

$\therefore$ x = 3

16. (B) Let the height be 'x'

$\therefore \text{Radius} = 1\frac{2}{3}x = \frac{5}{3}x$

Given $2\pi rh$ = 4620 cm2

$\Rightarrow 2 \times \frac{22}{7} \times \frac{5x}{3} \times x = 4620 \text{ cm}^2$

$x^2 = \cancel{4620}^{\cancel{210}\ 10521} \text{cm}^2 \times \frac{1}{\cancel{2}} \times \frac{7}{\cancel{22}_1} \times \frac{3}{\cancel{5}_1}$

$x^2 = (21\text{cm})^2$

$\therefore x$ = 21 cm

$\therefore \text{Radius} = \frac{5}{3}x = \frac{5 \times \cancel{21}^7 \text{cm}}{\cancel{3}_1} = 35\text{cm}$

Total surface area = $2\pi r$ (h + r)

$= 2 \times \frac{22}{\cancel{7}_1} \times \cancel{35}^5 \text{cm}(21 + 35)\text{cm}$

= 220 cm × 56cm = 12320 cm^2

17. (B) (a + b + c)2 – (a – b – c)2 = (a2 + b2 + c2 + 2ab + 2bc + 2ca) – (a2 + b2 + c2 – 2ab + 2bc – 2ca)

= a2 + b2 + c2 + 2ab + 2bc + 2ca – a2 – b2 – c2 + 2ab – 2bc + 2ca

= 4ab + 4ca

= 4a (b + c)

18. (B) LHS $\sqrt{\sqrt{3}-\sqrt{4-\sqrt{5}-\sqrt{12+5-2\sqrt{12}\times\sqrt{5}}}}$

$\sqrt{\sqrt{3}-\sqrt{4-\sqrt{5}-\sqrt{\left(\sqrt{12}-\sqrt{5}\right)^2}}}$

$= \sqrt{\sqrt{3}-\sqrt{4-\sqrt{5}-\left(\sqrt{12}-\sqrt{5}\right)}}$

$= \sqrt{\sqrt{3}-\sqrt{4-\sqrt{5}-\sqrt{12}+\sqrt{5}}}$

$= \sqrt{\sqrt{3}-\sqrt{4-\sqrt{12}}}$

$= \sqrt{\sqrt{3}-\sqrt{\left(\sqrt{3}\right)^2+1^2-2\sqrt{3}}}$

$= \sqrt{\sqrt{3}-\sqrt{\left(\sqrt{3}-1\right)^2}}$

$= \sqrt{\sqrt{3}-\sqrt{3}+1}$

$= \sqrt{1} = 1$

19. (A) Given $x^2 + x + 1 = 0$

$$\frac{x^2}{x}+\frac{x}{x}+\frac{1}{x}=0$$

$$x+1+\frac{1}{x}=0$$

$$x+\frac{1}{x}=-1$$

cubing on both sides

$$x^3+\frac{1}{x^3}+3x\times\frac{1}{x}\left(x+\frac{1}{x}\right)=-1$$

$$x^3+\frac{1}{x^3}+3(-1)=-1$$

$$x^3+\frac{1}{x^3}=-1+3$$

$$x^3+\frac{1}{x^3}=2$$

cubing in both sides

$$\left(x^3+\frac{1}{x^3}\right)^3 = 8$$

(OR) Given $x^2 + x + 1 = 0$

$(x - 1)(x^2 + x + 1) = 0\ (x - 1)$

$x^3 - 1^3 = 0 \Rightarrow x^3 = 1$

$$\therefore \left(x^3+\frac{1}{x^3}\right)^3=\left(1+\frac{1}{1}\right)^3$$

$= 2^3 = 8$

20. (A) Given $\angle A + \angle C = 140^\circ$

and $\angle A : \angle C = 1 : 3$

$\Rightarrow \angle A = 140^\circ \times \frac{1}{4} = 35^\circ$

and $\angle C = 140^\circ \times \frac{3}{4} = 35^\circ \times 3 = 105^\circ$

In the quadrilateral

$\angle A + \angle B + \angle C + \angle D = 360^\circ$

$\Rightarrow \angle B + \angle D = 360^\circ - (\angle A + \angle C)$

$= 360^\circ - 140^\circ$

$\therefore \angle B + \angle D = 220^\circ$

Given that $\angle B : \angle D = 5 : 6$,

$\angle B = 220^\circ \times \frac{5}{11} = 20^\circ \times 5 = 100^\circ$

and $\angle D = 220^\circ \times \frac{6}{11} = 20 \times 6 = 120^\circ$

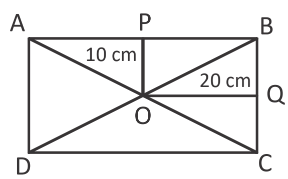
$\therefore$ The required angles are $\angle A = 35^\circ$, $\angle B = 100^\circ$, $\angle C = 105^\circ$ and $\angle D = 120^\circ$.

21. (D) BC = AD = 2PO = 20 cm

AB = DC = 2 × OQ = 40 cm

Perimeter of rectangle = 2(AB +BC)

= 120 cm



22. (C) Given $\left(\frac{d_1}{2}\right)^2 + \left(\frac{d_2}{2}\right)^2 = \left(\frac{34}{4}\text{cm}\right)^2$

$$\frac{d_1{}^2}{4} + \frac{d_2{}^2}{4} = \frac{289}{4}\text{cm}^2$$

$\therefore\ d_1^2 + d_2^2 = 289$

Given $d^1 + d^2 = 23$ cm

squaring on both sides

$d_1^2 + d_2^2 + 2d_1 d_2 = 529$

$289 + 2d^1 d^2 = 529$

$2d^1 d^2 = 240$

$$\frac{2d_1 d_2}{4} = \frac{240}{4}\text{cm}^2$$

Area of rhombus = $\frac{d_1 d_2}{2}$ = 60 cm^2

23. (A) Volumes ratio of sphere, cone & cylinder

$= \frac{4}{3}\pi r^3 : \frac{1}{3}\pi r^2 h : \pi r^2 h$

$= \frac{4}{3} : \frac{2}{3} : 2$

$= \frac{2}{3} : \frac{1}{3} : 1$

= 2 : 1 : 3

24. (C) Let the original radius be r cm

$\therefore$ Original surface area = $4\pi r^2$

New radius (R) = r + 100% r = r + r = 2r

New are surface = $4\pi R^2 = 4\pi(2r)^2$

= $4(4\pi R^2)$

Increased area = $4(4\pi R^2) - 4\pi r^2$

= $3(4\pi r^2)$

Increased area percentage

$= \frac{3(4\pi r^2)}{4\pi r^2} \times 100$ = 300 %

25. (C) Given $x + \frac{9}{x} = 6 \Rightarrow \frac{x^2 + 9}{x} = 6$

$\Rightarrow x^2 - 6x + 9 = 0$

$\Rightarrow x^2 - 3x - 3x + 9 = 0$

$\Rightarrow x = 3$

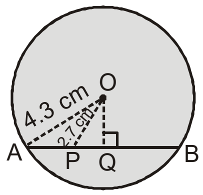
$\therefore\ x^2 + \frac{9}{x^2} = 9 + \frac{9}{9} = 9 + 1 = 10$

26. (D) $\Delta AOX \cong \Delta COY$ [$\because$ ASA congruency]

$\therefore$ OX = OY [$\because$ CPCT]

$\therefore$ OX – OY = 0

27. (B) As shown in the figure, OP = 2.7 cm and OA = 4.3 cm. Draw a perpendicular OQ to the chord AB. Clearly, AQ = QB. Since P divides AB in the ratio 7 : 10, let AP be 7x and PB be 10x.



Also, PQ = AQ – AP = $\frac{AB}{2}$ – AP

$= \frac{17x}{2} - 7x = 1.5x$

By Pythagoras' theorem, we have, $AQ^2 + OQ^2 = AO^2$.

Also, $OP^2 = PQ^2 + OQ^2$.

$\Rightarrow AQ^2 - PQ^2 = AO^2 - OP^2$

$\Rightarrow (8.5x)^2 - (1.5x)^2 = (4.3)^2 - (2.7)^2$

$\Rightarrow (10x)(7x) = 7(1.6)$

$\Rightarrow x^2 = 0.16$ or $x = 0.4$ cm.

$\therefore$ AB = $17x = 17 \times 0.4 = 6.8$ cm

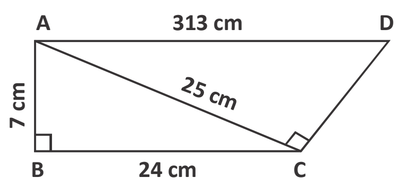
28. (B) $\angle RQP = 30^\circ + 225^\circ = 55^\circ$

$\angle RQP = +\ x = 180^\circ$

$55^\circ + x = 180^\circ$

$x = 180^\circ - 55^\circ = 125^\circ$

29. (D)



In ΔABC, $\angle B = 90^\circ$

$AC^2 = AB^2 + BC^2$

AB = 7 cm

In ΔACD, $\angle ACD = 90^\circ$

$AD^2 = AC^2 + CD^2$

$313^2 - 25^2 = CD^2$

97,969 – 625 = CD^2

CD = $\sqrt{97344}$

CD = 312 cm

Area of quad ABCD = Area of ABC + Area of ACD

$=\frac{1}{2}\times AB\times BC+\frac{1}{2}\times AC\times CD$

$=\frac{1}{2}\times 7\text{ cm}\times 24\text{ cm}+\frac{1}{2}\times 25\text{ cm}\times 312\text{ cm}$

= 84 cm^2 + 3900 cm^2

= 3984 cm^2

30. (C) ARPQ is a parallelogram

$\therefore$ AR = PQ & PR = AQ

$\therefore AR = PQ = \frac{AB}{2} = \frac{30cm}{2}$;

$PR = AQ = \frac{AC}{2} = \frac{21}{2}cm$

$\therefore AR + RP + PQ + QA$

$= \frac{30cm}{2}+\frac{30cm}{2}+\frac{21cm}{2}+\frac{21cm}{2}$

= 51cm

MATHEMATICS - 2 (MAQ)

31. (A,C,D) Given $x=\frac{1}{2-\sqrt{3}}\times\frac{2+\sqrt{3}}{2+\sqrt{3}}=2+\sqrt{3}$

$x-2=\sqrt{3}$

Squaring on both sides

$x^2 - 4x + 4 = 3$

$x^2 - 4x + 1 = 0$

$$x^2-4x+1)x^4-x^3+6x^2-17x+16(x^2+3x+5$$
$$x^4-4x^3+x^2$$
$$(-)\ (+)\ (-)$$
$$3x^3-7x^2-17x+16$$
$$3x^3-12x^2+3x$$
$$(-)\ (+)\ (-)$$
$$5x^2-20x+16$$
$$5x^2-20x+5$$
$$(-)\ (+)\ (-)$$
$$(11)$$

$$x^2-4x+1)x^3-2x^2-7x+2(x+2$$
$$x^3-4x^2+x$$
$$(-)\ (+)\ (-)$$
$$2x^2-8x+2$$
$$2x^2-8x+2$$
$$(0)$$

$$x^2-4x+1)x^4-x^3-6x^2-17x+5(x^2+3x+5$$
$$x^4-4x^3+x^2$$
$$(-)\ (+)\ (-)$$
$$3x^3-7x^2-17x+5$$
$$3x^3-12x^2+3x$$
$$(-)\ (+)\ (-)$$
$$5x^2-20x+5$$
$$5x^2-20x+5$$
$$(0)$$

32. (A,B,C) Given p(x) = $x^{2024} - y^{2024}$

p(y) = $(y)^{2024} - y^{2024}$

$= y^{2024} - y^{2024}$

p(y) = 0 ($x - y$) is a factor of p(x)

p($-y$) = $(-y)^{2024} - y^{2024}$

$= y^{2024} - y^{2024}$

p($-y$) = 0 ($x + y$) is a factor of p(x)

($x + y$) and ($x - y$) are factors of p(x)

($x^2 - y^2$) is also a factor of p(x)

33. (A,B) Given p(x) = $x^3q^2 - x^3pt + 4x^2pt - 4x^2q^2 + 3xq^2 - 3x\,pt$

$\therefore$ p(1) = q^2 – pt + 4pt – $4q^2$ + $3q^2$ – 3pt = 0

(x–1) is a factor of p(x)

p(3) = $27q^2$ – 27pt + 36pt – $36q^2$ + $9q^2$ – 9pt

= 0

$\therefore$ (x–3) is a factor of p(x)

34. (A,B,C)

Except option (D) all the lines don't pass through origin.

35. (A,C,D)

Square and rhombus are also parallelograms

<u>**REASONING**</u>

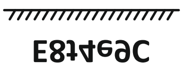
36. (C) Difference between the digits of 37 is 7 – 3 = 4. In the others, this rule is not satisfied.

37. (D)

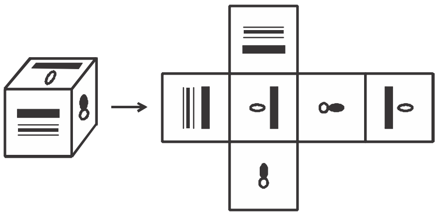
$7^2 - 5^2 = 24$	$22^2 - 20^2 = 84$
$5^2 - 3^2 = 16$	$11^2 - 9^2 = 40$
$6^2 - 4^2 = 20$	$9^2 - 7^2 = 32$
$3^2 - 1^2 = 8$	$\boxed{10}^2 - 8^2 = 36$

38. (C) GEYAAWT – GETAWAY means 'a quick departure'.

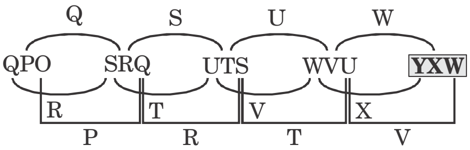
39. (D) **E8t4e9C**



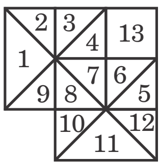
40. (A)



41. (C)



42. (C)



Small triangles → ⑫

2 + 3, 4 + 7, 7 + 6, 5 + 12, 10 + 8, 9 + 8 → ⑥

1 + 2 + 3, 8 + 10 + 11, 5 + 11 + 12, 1 + 8 + 9 → ④

2 + 3 + 4 + 7, 4 + 7 + 8 + 9, 6 + 7 + 8 + 10, 7 + 6 + 5 + 12 → ④

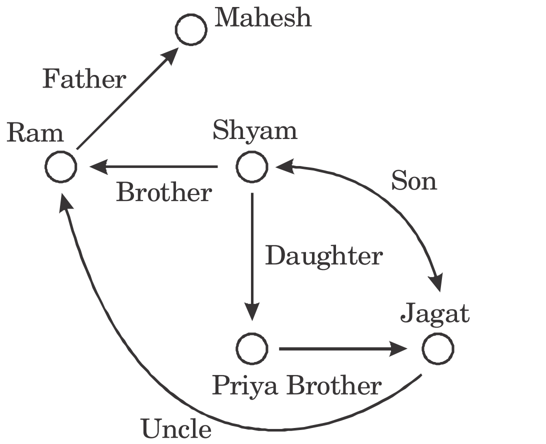
2 + 3 + 4 + 7 + 13 + 6 + 5 + 12 → ①

∴ Total number of triangles

= 12 + 6 + 4 + 4 + 1 = 27

Hence, there are 27 triangles.

43. (C) Jagat is the brother of Priya and Priya is the daughter of Shyam. Therefore Shyam is the father of Jagat. Ram is the brother of Shyam. Therefore, Ram is the uncle of Jagat.



44. (B) Shapes in the right diagonal interchange and the two shapes in the left top corner and right bottom corner interchange places and top corner gets a new shape.

45. (D) Code for white fill is G, and code for hexagon is R.

Hence, the code for  is **RG**.

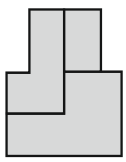
<u>**CRITICAL THINKING**</u>

46. (B) Assertion (A): True. Large-scale stubble burning in neighboring states like Punjab and Haryana contributes to high pollution levels in Delhi and NCR, particularly during October and November.

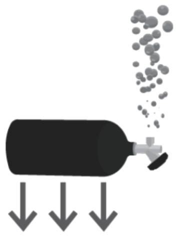
Reason (R): True. Burning of organic material releases pollutants, and stubble burning does contribute significantly to air pollution in Delhi. However, plume from stubble burning does not consistently account for 70% of total air pollutants. It fluctuates depending on weather and wind patterns, and other pollution sources like vehicles, industries, and dust also play a significant role.

Since both (A) and (R) are true but (R) is not the correct explanation for (A), the correct answer is (B).

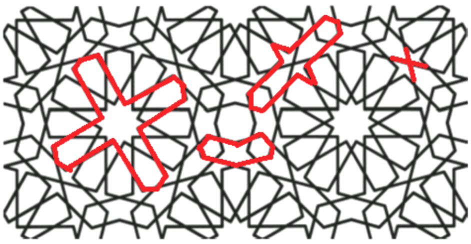
47. (C)



48. (D) As air escapes the available space is quickly replaced with water, so the tank' density becomes the same as that of the water and with the added weight and density of the tank itself continues to sink.



49. (B) (i), (iii), (v), (vi)



50. Delete